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ACCURACY AND CAPABILITIES OF THE  
ASC/IITRI CONIC SECTION TRAJECTORY SYSTEM

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1: ACCURACY AND CAPABILITIES OF THE  
ASC/IITRI CONIC SECTION TRAJECTORY SYSTEM

by

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and F. Narin

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The IITRI personnel who programmed various parts of the system are Mr. A. L. Friedlander, Mr. R. S. Hollitch, Mr. P. M. Pierce and Mr. F. Narin.

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## ABSTRACT

### ACCURACY AND CAPABILITIES OF THE ASC/IITRI CONIC SECTION TRAJECTORY SYSTEM

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The ASC/IITRI Conic Section Trajectory system was created to have a very flexible conic section trajectory capability to support the studies of scientific objectives, mission requirements, mission success probabilities and mission cost for solar system exploration under Contract NASr-65(06). This report discusses the accuracy and capabilities of the system and some of the studies which have been performed with the system. It shows that the planetary, asteroid, and comet orbital elements are adequate for conic section trajectory studies, and that the ASC/IITRI system results compare very closely to the JPL conic section results, and to the Mariner 2 flight to Venus. Transitions are shown to be smooth for elliptical to parabolic to hyperbolic trajectories. A complex launch hyperbolic excess speed vs. date of launch curve for 100 day flights to asteroid Eros is explained in detail to illustrate the physical meaning behind the somewhat complex energy requirements of interplanetary flight. The options available in the ASC/IITRI system for flights to points anywhere in the solar system are outlined. Finally, an annotated listing of programs and subroutines is included.

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ACCURACY AND CAPABILITIES OF ASC/IITRI  
CONIC SECTION TRAJECTORY SYSTEM

1. INTRODUCTION

This report discusses the accuracy and capabilities of the ASC/IITRI Conic Section Trajectory system, and supersedes a previous report (Narin, Pierce, Schmidt 1963) summarizing the system.

The ASC/IITRI Conic Section Trajectory system is designed to provide, in a fully automatic manner, ballistic trajectory calculations from any one of the 3800 objects whose orbital elements are on our orbital elements tape to any other object; the system also provides the capability of using its subsections for other calculations. The standard conic section trajectory equations in the three dimensional solar system are evaluated on the IBM 7090 computer in FORTRAN II language; simplicity, flexibility and ease of use were the guiding factors in devising the system. All bodies are assumed to be in elliptical, hyperbolic or parabolic orbits.

The main trajectory calculation proceeds as follows:

Given time of launch from one body and time of arrival at a second, the computer calculates VHL, the launch hyperbolic excess speed,  $\Delta V$  the ideal velocity, VHP the hyperbolic excess speed at the target, the parameters of the spacecraft trajectory, and a number of additional

quantities defined in Section 4 of this report. Launches are restricted to the direction of the Earth's orbital motion.

An optional output is the positions of Earth, target, spacecraft and any other three objects throughout the flight. All standard outputs may be plotted, as an option, on the IBM 1401 printer.

Calculation time, without IBM 1401 plotting or other options, is about five trajectories per second of IBM 7090 time for the main code.

In addition to the main code and its subroutines there are codes for near planet maneuvers, for determining if a comet or asteroid trajectory will be perturbed by a close approach to a planet, for determining the number of hours in any day an object would be visible from a ground based observatory, for surveying and analyzing object positions and orbital elements and for doing a variety of subsidiary calculations. Section 2 of this report discusses some of the studies performed with the system.

Section 3 of this report outlines the steps taken to assure the validity of the target orbital data on our orbital elements tape, and Section 4 compares in detail some ASC and JPL calculations and defines the quantities calculated by the codes. Section 5 briefly discusses the absolute accuracy question, and the Mariner 2 flight to Venus. Section 6 compares elliptic, hyperbolic and parabolic trajectories. Section 7 then explains in detail the features of a somewhat complex VHL vs. launch date curve, for the asteroid Eros, to illustrate the physical meaning behind the somewhat complex energy requirements of interplanetary flight. Section 8 discusses the special target options which allow the system to be used to calculate trajectories from Earth to any place in the solar system. Section 9 then outlines the



various supplementary and auxiliary programs, and the important sub-routines which make up the system.

## 2. STUDIES PERFORMED WITH THE SYSTEM

### 2.1 Jovian Missions

In support of a Jovian Mission Study (Stone et al, 1963) extensive computations were made of the energy requirement for one-way ballistic trajectories to Jupiter in the 1970 to 1975 time period. Typical curves related date of launch to ideal velocity, communications distance and hyperbolic excess speed at Jupiter for various spacecraft times of flight.

### 2.2 Solar System Summary

A survey of one-way ballistic trajectory data (Narin and Pierce, 1964) was performed for flights from Earth to various solar system targets. Typical curves related ideal velocity to time of flight for minimum energy flights to the Planets, to points in and out of the ecliptic plane, and to angles out of the ecliptic plane. There was also extensive coverage of the energy requirement for transfer from planet approach hyperbolas into orbits around the planets.

### 2.3 Trajectories to the Comets

An extensive survey of ballistic trajectories to the short period comets (Pierce and Narin, 1964) was carried out to delineate the comets which appear to be good targets in the 1965 to 1975 time periods. This study included the determination of time of flight, communication distance and hyperbolic excess speed at the comet for minimum energy launches to the comets, all as a function of launch date. The recovery and sighting

problems for determining when the comet would be visible from Earth were considered, as were determining if a comet would be perturbed in its orbit. Consideration was also given to guidance problems.

#### 2.4 Accessible Regions Method

A method was devised (Narin, 1964) for plotting contours of ideal velocity and time of flight in the solar system, thus delineating the regions of the solar system accessible to various launch vehicles.

#### 2.5 Asteroid Spatial Distribution

A survey was performed (Narin, 1964) to see if there were any non-uniformities in the spatial distribution of the asteroids in the asteroid belts, to ascertain if some regions of the belt had a higher density of asteroids at a given time than other regions. No non-statistical fluctuations of any significance were found. This study included the computation of the positions of 2000 asteroids over a 30 year period (1965-1995).

### 3. ACCURACY OF ORBITAL PARAMETERS

The orbital elements tape used with the ASC/IITRI system is a FORTRAN II BCD magnetic tape containing the orbital elements and names of 3800 solar system objects. There are elements on the tape for three types of solar system objects, planets, asteroids and comets, in two forms. The orbital elements of the planets are heliocentric, referred to the ecliptic plane and the mean equinox of date, and time dependent. The comets and asteroids are heliocentric, referred to the ecliptic plane and the mean equinox of 1950.0, and time independent. The following explains the checking methods and results for each type of body.

### 3.1 Planet Data

The planetary data (Allen, 1955) are mean, time dependent orbital parameters corrected for long-term perturbative effects. The 7090 calculated positional data, based on Allen's data, were checked against "Planetary Coordinates" data (H. M. Nautical Almanac Office, 1961).

A table was prepared of the Planetary Coordinates data minus the 7090 calculated data for heliocentric longitude, latitude, and radius over the twenty year span. Table 1 lists the maximum differences in longitude, latitude, and radius encountered in the 1960-1980 period for the planets Venus through Neptune. Mercury and Pluto are considered separately. The errors in latitude and radius are minute as evidenced by Table 1. The differences in longitude are also small and attributed to short-term perturbations; Saturn has the largest longitude deviation of less than 0.17 degrees. From the viewpoint of conic section trajectory calculations these differences are completely negligible.

Figure 1 shows the difference between the calculated and Planetary Coordinates data for Jupiter from 1960-1980; it was chosen as representative of all the planets. For Mercury and Pluto the differences are perhaps significant. The 7090 calculations for Mercury oscillate in heliocentric longitude with an amplitude of about four degrees about the Planetary Coordinate's figures, while Pluto shows a gradual deterioration over the time span, the worst of which is 2.7 degrees in heliocentric longitude. In Section 4 some ASC/IITRI and JPL calculated flights to Mercury will be considered; this comparison shows that the ASC results are very similar to JPL's.

Table 1

MAXIMUM DIFFERENCE BETWEEN THE "PLANETARY  
COORDINATES" DATA AND THE ASC 7090 CALCULATED DATA  
OVER THE PERIOD 1960-1980

	Heliocentric Longitude (degrees)	Heliocentric Latitude (degrees)	Heliocentric Radius (AU)
Venus	.037	.004	0.0001
Earth	.058	.002	0.0001
Mars	.106	.004	.0001
Jupiter	.081	.004	.003
Saturn	.170	.007	.009
Uranus	.032	.004	.012
Neptune	.095	.002	.029

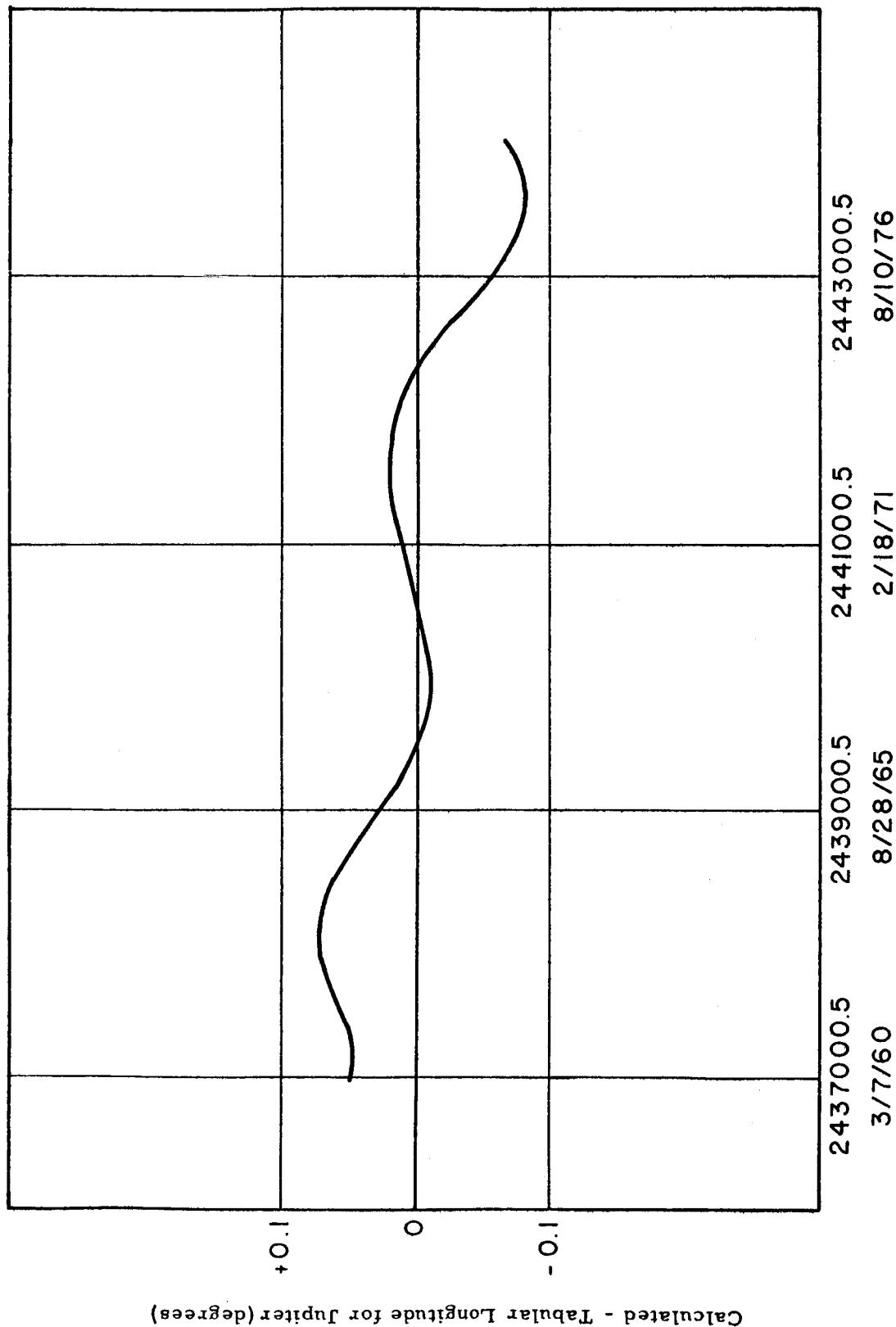


Figure 1 IBM 7090 Calculated Longitude for Jupiter Minus "Planetary Coordinates For the Years 1960-1980" Tabulated Longitude Vs. Date

### 3.2 Asteroid Data

The data for 1563 numbered asteroids (Cincinnati Observatory, 1948) was checked against the edition for 1964 of the American Ephemeris (Nautical Almanac Office, 1962) which contains day to day positional data for the asteroids Ceres, Pallas, Juno, and Vesta in geocentric equatorial coordinates of right ascension, declination, and Earth-object distance.

Positional elements for the asteroid Ceres were computed by the 7090 and checked against the American Ephemeris for various times using four significant figures, with rotation and translation from heliocentric ecliptic coordinates by a hand calculation. The results are adequate for conic section calculations, as demonstrated by the following typical example.

Date of check - July 1, 1964

	<u>7090 Calculated</u>	<u>American Ephemeris</u>
Right Ascension	17 <sup>h</sup> 48 <sup>m</sup> 27 <sup>s</sup>	17 <sup>h</sup> 49 <sup>m</sup> 54.71 <sup>s</sup>
Declination	-26° 32' 58"	-26° 38' 51"
Earth-Ceres Distance (AU)	1.859	1.860533

The errors are due to two factors: first, elements from the American Ephemeris have been adjusted for perturbations and recent observations while Cincinnati data was published in 1948, and second, American Ephemeris carried eight places of accuracy while ASC coordinate transformations were calculated to four places.

Because each asteroid is handled by the computer codes in exactly the same manner, in effect the system is checked out from the

results of Ceres. The exception could be a typographical or keypunch error in the original processing of the data.

Orbital data for an additional 2000 unnumbered asteroids are also on the orbital elements tape (Cincinnati Observatory, 1961).

### 3.3 Comet Data

The data for 80 periodic comets (Porter, 1961) was compared with data in an STL study (Space Technology Laboratory, Inc., 1963). Perihelion times for the comet Encke were cross-checked against STL and they agreed exactly to the one day accuracy given in the report. The longitude of perihelion also was checked and agreed with their  $185.2^\circ$ . Additional extensive comparison of ASC and STL calculations have been done for a few tens of comets, with the expected close agreement.

Because ASC and STL used the same sources for their comet data, indication is given that no systematic errors have been introduced by the computer codes.

## 4. ASC-JPL CONIC SECTION COMPARISON

A comparison of a single (randomly picked) JPL conic section flight and the same flight as calculated by ASC/IITRI shows complete agreement for the parameters which are common to both systems. To ease the use of the ASC/IITRI system the nomenclature and definitions of JPL have been adopted as completely as possible. Table 2 shows JPL and ASC/IITRI nomenclature, and JPL definitions. Table 3 shows numbers calculated by the two systems. The case is a 170 day flight, Earth to Venus, launch 9/14/65, arrival 3/3/66 (Clarke, Jr., 1963). Many other ASC-JPL comparisons have been made with the same close agreement.

Table 2

ASC/IITRI AND JPL NOMENCLATURE

ASC/IITRI	JPL	
VHL	VHL	the launch hyperbolic excess speed
VHP	VHP	the hyperbolic excess speed at the target
RL	$R_L$	the heliocentric radius of the launch planet at time of launch
LAL	LAL	the celestial latitude of the launch planet at time of launch
LOL	LOL	the celestial longitude <sup>1</sup> of the launch planet at time of launch
VL	$V_L$	the heliocentric speed of the probe at time of launch
HCA	HCA	the heliocentric central angle, the angle between the position vector $R_L$ of the launch planet at time of the launch and the position vector $R_p$ of the target planet at time of arrival of the spacecraft
A	SMA, A	the semi-major axis of the heliocentric transfer-ellipse
ECC	ECC, e	the eccentricity of the heliocentric transfer ellipse
INC	INC, i	the inclination of the heliocentric transfer ellipse
V1	V1	the heliocentric speed of the launch planet at time of launch
RP	$R_P$	the heliocentric radius of the target planet at time of arrival

<sup>1</sup> JPL uses mean equinox of date while ASC/IITRI uses equinox of 1950.0 as reference direction.



Table 2 (Cont'd)

ASC/IITRI	JPL	
LAP	LAP	the celestial latitude of the target planet at time of arrival
LOP	LOP	the celestial longitude of the target planet at time of arrival
VP	VP	the heliocentric speed of the probe at time of arrival
TAL	TAL	the true anomaly of the probe in the heliocentric transfer ellipse at time of launch
TAP	TAP	the true anomaly of the probe in the heliocentric transfer ellipse at time of arrival
V2	V <sub>2</sub>	the heliocentric speed of the target planet at time of arrival
RC	RC	the communication distance, or distance between the launch planet and the target at time of arrival of the spacecraft at the target

Additional IITRI output are:

RS, MN	minimum spacecraft/sun distance throughout the flight
HCA, 2	the heliocentric central angle between the position vector of the launch planet at time of arrival of the spacecraft at the target and the position vector of the target at the same time
$\Delta V$	the ideal velocity, that is, the total velocity increment which must be given to the spacecraft on leaving Earth:

$$\Delta V = \sqrt{(36,178)^2 + (VHL)^2} + 4000$$

Here 36,178 ft/sec is the characteristic velocity for Earth escape launching from Cape Kennedy and 4000 ft/sec is a correction for gravitational and frictional losses during launch

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Table 2 (Cont'd)

ASC/IITRI	
DV	the velocity increment required to transfer a spacecraft from its hyperbola into an orbit around a target
$\phi_{SL}$	the angle between the velocity of the spacecraft at launch and the velocity of the Earth at time of launch
$\phi_{SP}$	the angle between the velocity of the spacecraft at arrival at the target and the velocity of the target at time of arrival
$\phi_{SPL}$	the angle between the velocity of the spacecraft at launch and the plane of the Earth's orbit
$\phi_{SPP}$	the angle between the velocity of the spacecraft at arrival at the target and the plane of the target's orbit
$\phi_{LCHLIM}$	the range of azimuth's at Cape Kennedy from which a direct launch into the plane of the transfer orbit cannot be made

Table 3

ASC-JPL COMPARISON FLIGHT:  
EARTH TO VENUS  
LAUNCH 9/14/65, ARRIVAL 3/3/66

Quantity	JPL No.	IITRI No.
VHL	7.958 km/sec	8.0
VHP	9.994 km/sec	10.0
$C_3$ ; in units $\text{km}^2/\text{sec}^2$ , $C_3$ is simply $(\text{VHL})^2$ .		
$R_L$	$150.47 \times 10^6 \text{ km}$	$151 \times 10^6$
LAL	0	0
<sup>1</sup> LOL	350.97 deg	350.76
$V_L$	27.315 km/sec	27.313
HCA	193.24 deg	193.24
A	$139.37 \times 10^6 \text{ km}$	$139.4 \times 10^6$
ECC	.17566	.1757
INC	14.2801 deg	14.29
V1	29.612 km/sec	29.611
RP	$107.76 \times 10^6 \text{ km}$	$108 \times 10^6$
LAP	3.24 deg	3.24
<sup>1</sup> LOP	183.82 deg	183.60
VP	38.015 km/sec	38.016
TAL	155.85 deg	155.85

<sup>1</sup> The differences in longitude are the result of ASC/IITRI equinox of 1950.0 reference direction, and JPL's equinox of date reference.

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Table 3 (Cont'd)

Quantity	JPL No.	IITRI No.
TAP	349.09 deg	349.09
$V_2$	35.166 km/sec	35.166
RC	$63.151 \times 10^6$ km	$63 \times 10^6$

As a further check on the ASC/IITRI system, and particularly on the orbital data for Mercury, the points shown as crosses on Figure 2 were calculated and plotted on the JPL curve. The agreement is more than adequate; the small deviations are due to interpolation and rounding in the computer output. This type of curve is generated by computing a series of trajectories for various times of flight for each launch date, and determining the minimum energy flight on that day.  $C_3$  is simply  $VHL^2$ .

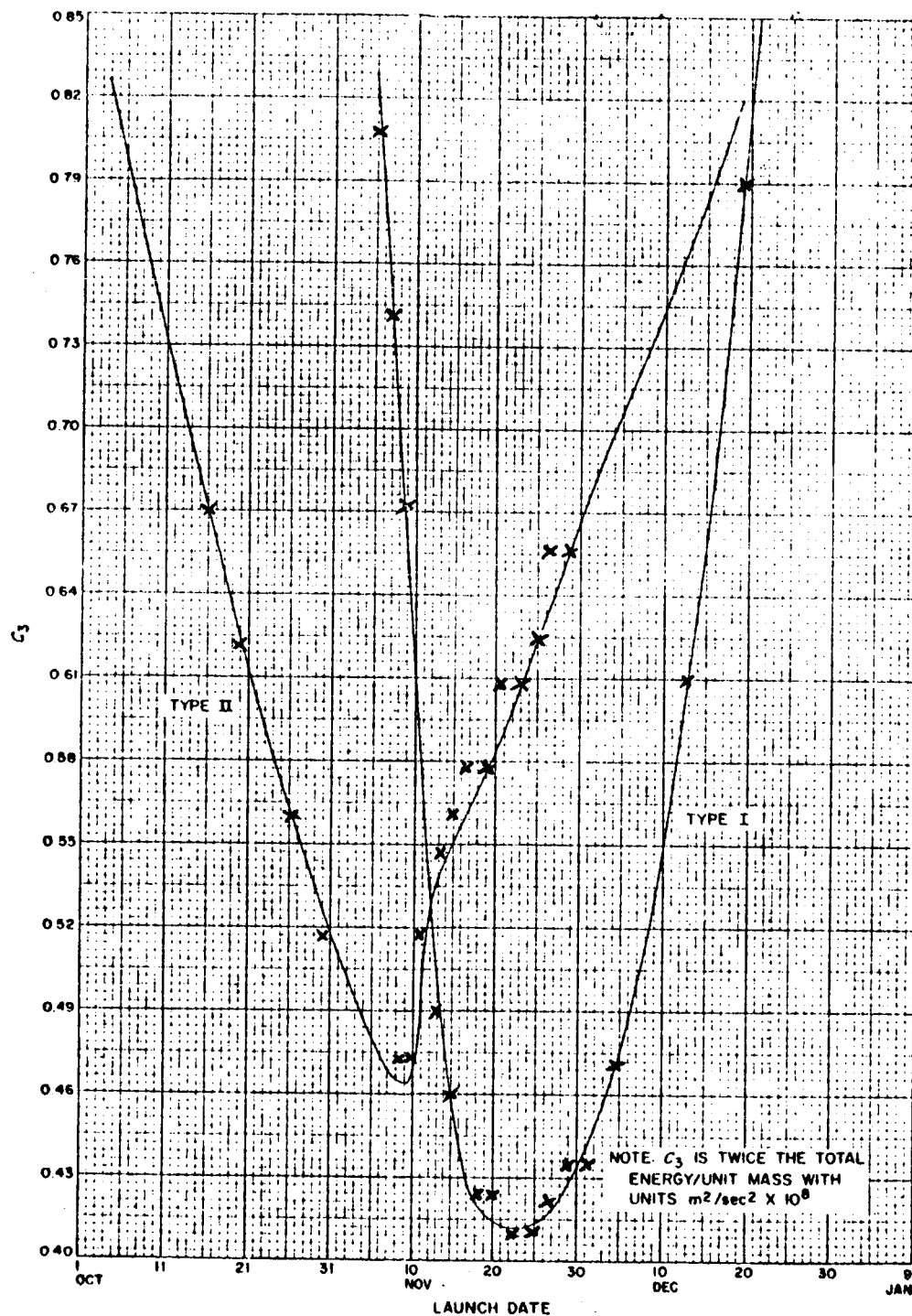


Figure 2 **Fig. 17. Mercury 1967: Minimum injection energy vs launch date**  
ASC/IITRI Calculated Points for 1967 Flights to  
Mercury Plotted on JPL Curve

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5. ACCURACY OF CONIC SECTION TRAJECTORIES

Since we do not yet have a high precision trajectory program with which to compare the conic section system, this measure of the accuracy of the system is not available. However, since our system is comparable with JPL's the following may be quoted from a JPL report (Clarke, 1963).

"Extensive accuracy studies were performed to verify the adequacy of these trajectories for preliminary design use. Both Mars and Venus trajectories were computed on the JPL precision-integrating trajectory program using initial conditions obtained from the approximate trajectories contained herein. Of 56 Mars cases run, 29 missed the target by less than 500,000 km; 16 missed by between 500,000 and 1,000,000 km; and 5 missed by between 1,000,000 and 1,500,000 km. The worst case missed by 3,500,000 km. For the flight time errors, 16 varied between 1 and 2 days; 14 varied between 2 and 3 days; and 9 were greater than 3 days. The worst case was 7.2 days. No systematic properties of these errors were noted except that they appear to get worse for the higher-energy trajectories.

For Venus, the accuracy was considerably better, averaging 322,000 km miss error and 0.67 day flight time errors. Based on these comparisons, the model used to generate the trajectories contained herein is considered to be adequate and the results suitable for preliminary mission design studies. These results are very useful for initializing the precision trajectory search program.

When used for the stated purposes, these trajectories provide an excellent source of data obtained at considerably less time and expense than precision cases."

One very interesting check on the system was obtained by running the Mariner 2 flight to Venus. The Mariner 2 launch occurred on August 27, 1962 with an Atlas-Agena B vehicle carrying the 447 pound payload to the planet. The flight time was 109 days. For a 109 day flight, Earth to Venus, launching on 8/27/62, the ASC/IITRI codes computed an ideal velocity  $\Delta V$ ; using this  $\Delta V$  and a typical Atlas-Agena performance curve (Nelson, 1963) we compute a possible payload weight of 380 pounds; the difference

of 67 pounds of payload corresponds to a  $\Delta V$  difference of only 350 ft/sec. Since there are variations from one Atlas-Agena to another the main usefulness of this calculation is in providing a qualitative impression of the accuracy of this type of computation.

6. TYPES OF CONIC SECTION TRAJECTORIES

The ASC/IITRI system will calculate parabolic, hyperbolic, or elliptical trajectories in a completely automatic manner. The following is an example of a series of trajectories which show all three types.

Table 4

DIFFERENT TYPES OF CONIC SECTION TRAJECTORIES:  
400 DAY FLIGHTS, EARTH TO JUPITER  
LAUNCHES EVERY 5 DAYS

Launch Date	VHL (km/sec)	VHP (km/sec)	Type of Trajectory	HCA (degrees)	RC (AU)	ECC
9/14/77	12.6	17.1	Hyperbolic	125.1	5.35	1.020
9/19/77	12.7	16.9	Hyperbolic	120.6	5.28	1.010
9/24/77	13.1	16.7	Parabolic	116.2	5.20	1.000
9/29/77	13.7	16.5	Elliptical	111.7	5.12	0.9895
10/4/77	14.4	16.3	Elliptical	107.1	5.04	0.9789
10/9/77	15.3	16.0	Elliptical	102.6	4.97	0.9685

A glance at the VHL or VHP values shows the smooth transition from one type of trajectory to the other.



7. EXPLANATION OF A VHL Vs. DATE OF LAUNCH CURVE

In order to illustrate the physical meaning behind the somewhat complex energy requirements of interplanetary flight we have included this section on flights to the asteroid Eros.

Figure 3 shows the launch hyperbolic excess speed VHL required for 100 day flights to Eros for launches in 1965 to 1969. The orbital parameters for Eros are the following (Cincinnati Observatory, 1948).

Table 5

ORBITAL PARAMETERS FOR EROS

A = 1.46 AU	(semi-major axis)
ECC = .2228	(eccentricity)
INC = 10.8 degrees	(inclination)
$\Omega$ = 304. degrees	(longitude of the ascending node)
$\bar{\omega}$ = 122. degrees	(longitude of perihelion)
T = 1.76 sidereal years	(period)
Perihelia = 4/4/66, 1/7/68, 10/11/69	
Perihelion distance = 1.333 AU	

Figure 4 defines this coordinate system and Figure 5 shows Earth and Eros heliocentric longitudes. The maximum heliocentric latitude of Eros is  $\pm 11$  degrees; this is ignored in Figure 5. It should be noted that perihelion occurs very close (2 degrees) to the descending node of the orbit, and aphelion close to the ascending node. Thus flights to Eros arriving near perihelion will be low energy both because of the relatively short flight distance and because they can be flown in or very nearly in the ecliptic plane. Figure 5 shows the Earth position 100 days before each of the perihelia in 1965-1969. For launch in 1967 the 100 day flight is reasonably

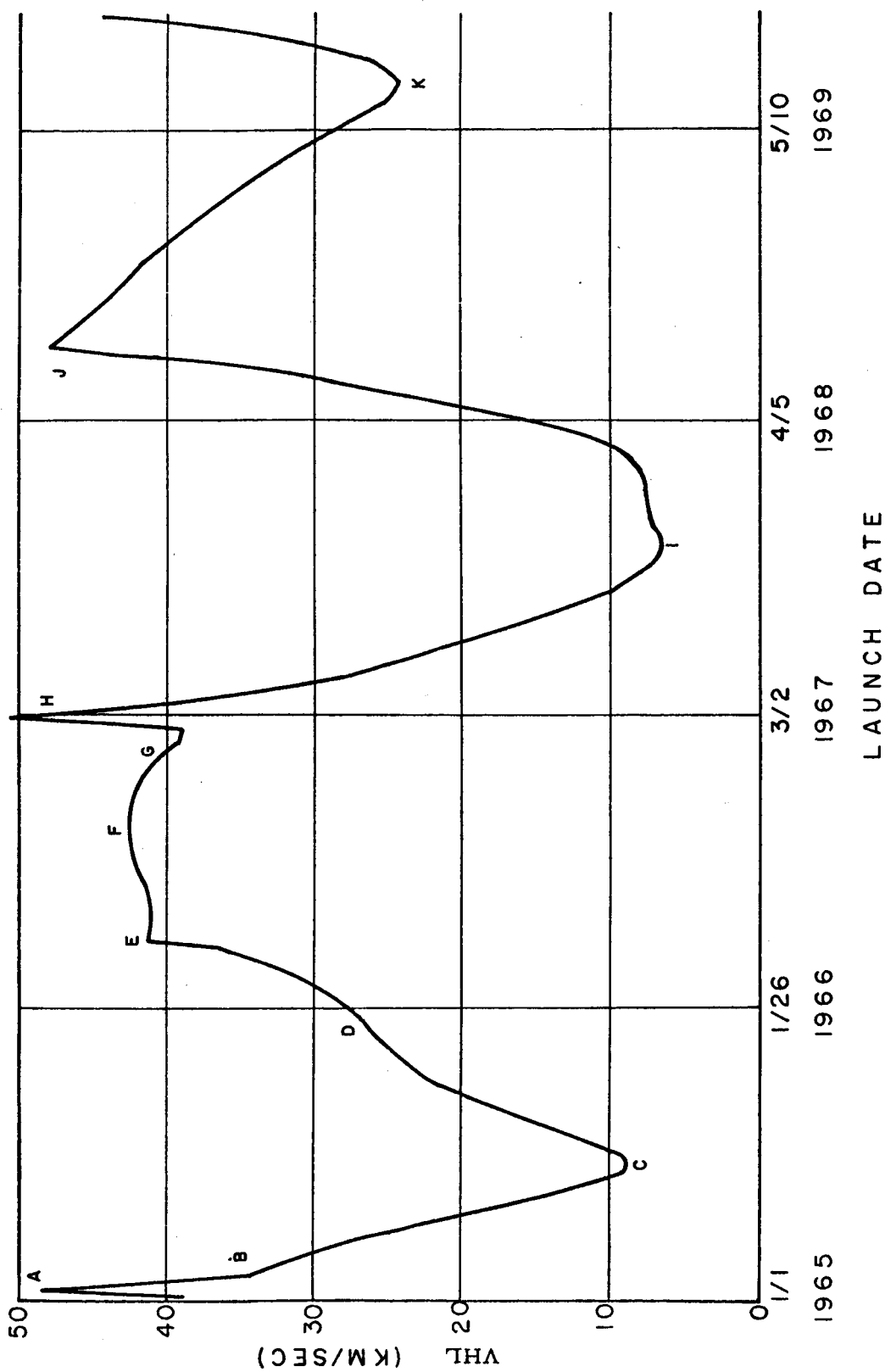


Figure 3 Launch Hyperbolic Excess Speed Required for 100 Day Flight to Asteroid Eros

Vs. Launch Date, 1965 to 1969

### Nomenclature

$\Omega$  = longitude of the ascending node  
 $\varpi$  = longitude of perihelion  
 $\omega$  = argument of perihelion  
 $u$  = argument of latitude  
 $\eta$  = true anomaly  
 $\beta$  = heliocentric latitude  
 $\lambda$  = heliocentric longitude  
 $i$  = inclination

### Useful Relations

$$\lambda = \Omega + \tan^{-1}(\cos i \tan u)$$

$$\sin \beta = \sin u \sin i$$

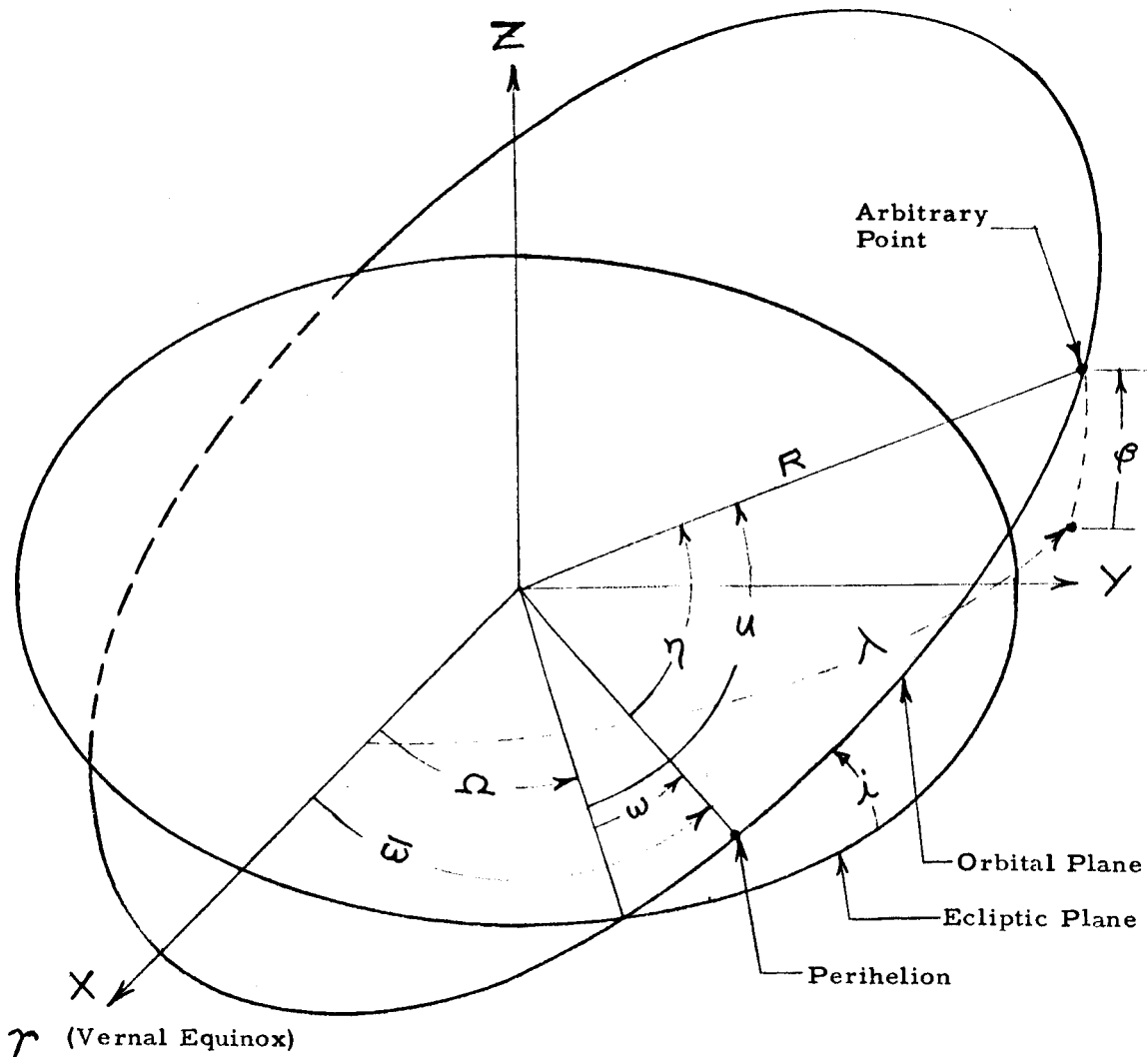


Figure 4 Heliocentric, Ecliptic Coordinate System

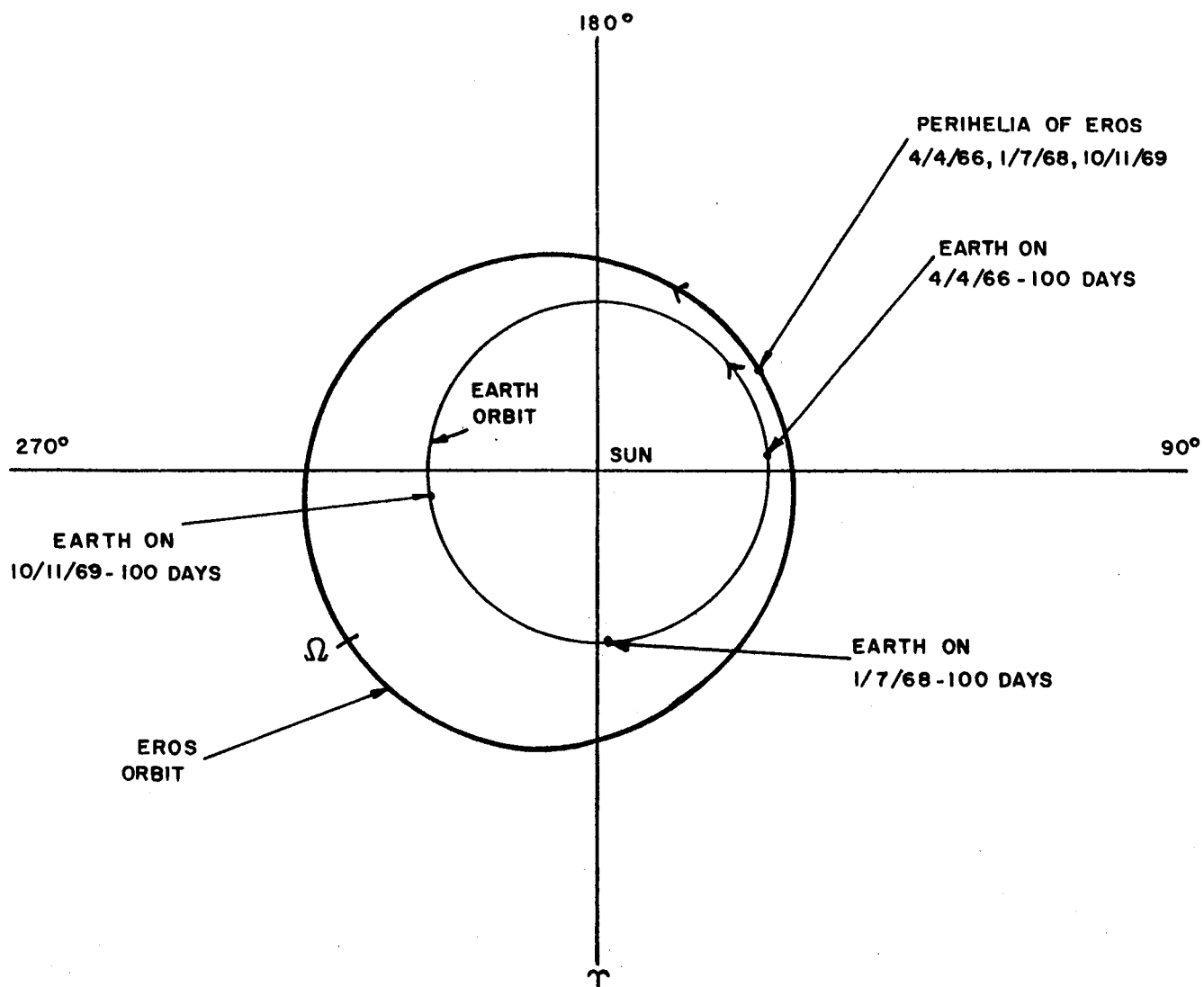


Figure 5 ORBITS OF EARTH AND ASTEROID EROS (LONGITUDES PLOTTED ON ECLIPTIC PLANE)

favorable, with VHL of 9.5 km/sec, while for the 1966 and 1969 launches the Earth is, respectively, too near and too far from Eros at 100 days before perihelion; VHL for both these flights is about 25 km/sec. Since the Earth moves at approximately 1 degree per day a low energy flight to Eros would be one with maximum utilization of the Earth's orbital velocity, that is, the Earth would be approximately 100 degrees behind Eros in longitude 100 days before perihelion. For the 1967 launch this condition is missed by about 20 days.

Consider Figure 3 again. The first peak "A" corresponds to a launch at 1/11/65 for a flight with heliocentric central angle HCA of 183 degrees; that is, the flight path is highly inclined and the spacecraft must fly nearly over the Sun. The required VHL then decreases as the HCA decreases; the change of slope "B" corresponds to flights arriving at Eros near aphelion and before and after Eros passes through the ecliptic plane at the ascending node of its orbit. The required VHL continues to decrease for later launches as the HCA decreases; however Eros is now ascending above the ecliptic. The first minimum "C" is reached for launches in July 1965 arriving at Eros in October, with an HCA of 80 degrees, when Eros is 9 degrees above the ecliptic and 1.6 AU from the Sun. As HCA gets smaller the required VHL rises, with the flight "D" launching 12/27/65 and arriving near perihelion having a high VHL of 26 km/sec because the HCA is only 28 degrees. The apparent cusp "E" corresponds to the Earth (at launch) passing ahead of Eros (at arrival) and the HCA going from 0 to 360° (since the code restricts launches to the direction of the Earth's orbital motion). The flights "F" are unfavorable, with large HCAs and Eros considerably below the ecliptic. The required VHL decreases

slightly at "G" as Eros again nears the ecliptic plane and aphelion, but this decrease is terminated by the high energy spike "H" which corresponds to a highly inclined flight path of 180 degrees HCA. After this the HCA decreases, pushing the VHL requirement down, even while Eros is rising out of the ecliptic. The minimum energy occurs for a flight "I" launched October 1967 with an HCA of 113 degrees, arriving about a month after perihelion when Eros is 3 degrees below the ecliptic; the flight arriving at perihelion launches a month before "I" but has higher energy required because the HCA is slightly greater. The next peak "J" corresponds to an HCA of 360 degrees, that is, the Earth (at launch) is again passing Eros at its arrival position. The last minimum on the curve, "K", corresponds to a flight, arriving at Eros at its perihelion in 1969, with relatively unfavorable HCA of 200 degrees. Thus this complex curve reflects the interactions as Eros and the Earth move in their orbits. Each maxima, minima and inflection point can be shown to correspond to a specific geometric configuration.

## 8. SPECIAL TARGET OPTIONS

In order to calculate parameters for flights to objects or places not on the ephemeris tape there are various options in the conic section system, one using the regular main code BUMP, and others using the special codes BUMP-X and BUMP-XC, BUMP-OC and BUMP-OG.

### 8.1 Special Target Option on BUMP

When using the special target option in BUMP a set of orbital parameters are fed in for a target and the system proceeds normally thereafter. Note that this requires that the target be moving in a conic section trajectory, and that by fixing the semi-major axis of the conic (if it is an ellipse) one fixes the period. For flights to particular positions out of the ecliptic plane this system is awkward, since the target is moving in a plane inclined to the ecliptic and thus its position varies with time. To circumvent this difficulty modifications to BUMP were made to fix the target position; these versions are BUMP-X and BUMP-XC.

### 8.2 BUMP-X and BUMP-XC

Bump-X and BUMP-XC are identical codes except that in BUMP-X the Earth is assumed to be in its normal elliptical orbit (eccentricity = .017) while in BUMP-XC the Earth's orbit is set as circular. In either case one specifies the distance from the Sun  $R$  of the target and its elevation  $Z$  normal to the ecliptic plane; the codes then calculate the flight parameters for a series of flights from Earth to the position  $R$ ,  $Z$  by assuming this position fixed at some heliocentric longitude and moving the Earth around in its orbit in 10 degree increments.

### 8.3 BUMP-OC and BUMP-OG

These two versions of BUMP are identical except for input features; they set the Earth's orbit as circular, and are optimized in the sense that for each target R, Z and each time of flight, the Earth is moved in its orbit to find the minimum energy Earth- target geometry. The Earth is moved first in 30 degree steps, then in 5 degree steps to determine the minimum energy launch geometry to within 5 degrees. These versions of BUMP have an abbreviated output and are about three times faster than the others.



9. ANNOTATED LIST OF PROGRAMS AND SUBROUTINES

BUMP: BUMP is the main supervisory program which controls input, output and the flow of computations, and performs some of the simpler calculations and scaling for output.

BUMP-X, XC,  
OC, OG These are versions of the BUMP supervisory program which calculate the parameters for flights to specified places in the solar system.

DATE: DATE is a subroutine which finds the date from a given time in seconds after 1950.0 U. T.

FDPLNE: FDPLNE is a subroutine which finds the plane in which two radius vectors lie.

GMST: GMST is a program which calculates the Greenwich mean sidereal time at 0 hours U. T. of that date.

GUIDE: GUIDE is a program, based on linear perturbation and covariance methods, which computes conic trajectory sensitivity to injection velocity errors and mid-course velocity correction requirements. Numerical results are expressed in terms of RMS statistical averages.

JULTIM: JULTIM is a subroutine which finds the Julian date from the calendar date.

KEPLER: KEPLER is a subroutine which solves Kepler's transcendental equation for elliptical or hyperbolic trajectories.

LMCON: LMCON is a subroutine which finds the parameters of the spacecraft orbit, given the plane from FDPLNE, the Earth and target positions and the time of flight. LMCON is based largely on a JPL memo by V. C. Clarke, Jr.

LOCATE: LOCATE is a subroutine which takes the orbital parameters for a body in an orbit, and the time, and finds the position, angles, velocities, etc. of the body.

LUMPER: LUMPER is a program for grouping the positions of large numbers of asteroids, at a given time, to see if there are clusters of asteroids in the distribution of asteroids in the asteroid belt.

**NRMSS:** NRMSS is a program for determining if a given object's trajectory will be perturbed by the planets, by surveying the relative positions, over a time span, of the planets and other objects in the solar system. The code calculates planet to object distances, object to Sun distances, and the magnitudes of the solar and planetary gravitational forces at the objects.

**OESORT:** OESORT is a program which lists the asteroids and comets on the orbital elements tape in order of increasing semi-major axis, eccentricity, inclination, period, perihelion distance or time of perihelion.

**PLDELV:** PLDELV is a trio of programs which calculate the velocity increment required to transfer a spacecraft, approaching a planet in a hyperbolic orbit with respect to the planet, into an elliptical, parabolic or circular orbit around the planet.

**SCAN:** SCAN is a program which calculates the heliocentric position, at a given time, of any object on the orbital elements tape and also the relative positions of the body and the Earth.

**SIGHT:** SIGHT is a program for determining the number of hours a day an object will be visible from an Earth observatory. This program is used primarily in considering optical recovery of comets.

**SWITCH:** SWITCH is a program which effects coordinate system transformations from heliocentric ecliptic to geocentric equatorial.

**TIME:** TIME is a subroutine which finds the time in seconds after 1950.0 U. T. from a given date.

**UPDATE:** UPDATE is a program which adds new data to the ephemeris tape, either to update orbital parameters on the tape or to add new objects to the tape.

**ORBITAL ELEMENTS TAPE:** ORBITAL ELEMENTS TAPE is a BCD FORTRAN II tape with 2 records for each object on the tape. The ephemeris tape currently has positional data for the Sun and planets, 90 periodic comets and 3700 asteroids. These data are a set of 7 orbital parameters describing the orbits of the bodies; all orbits on the tape are elliptical or circular although there is no reason why hyperbolic or parabolic orbits could not be added. For the comets and asteroids the orbital parameters are heliocentric, ecliptic, equinox of 1950.0 and constant; for the planets the parameters are heliocentric, ecliptic, equinox of date and varying with time.

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